Calculus II

Series - Things to Consider

Important: This cheat sheet is not intended to be a list of guaranteed rules to follow. This introduces some hints and some ideas you may consider when choosing tests for convergence or divergence when evaluating a given series.

It is usually a good idea to try using the **Test for Divergence** as a first step when evaluating a series for convergence or divergence. If we can show that:

$$\lim_{n\to\infty} a_n \neq 0$$

Then we can say that the series diverges without having to do any extra work.

Below are some general cases in which each test may help:

P-Series Test:

- The series be written in the form: $\sum \frac{1}{n^p}$

Geometric Series Test:

- When the series can be written in the form: $\sum a_n r^{n-1}$ or $\sum a_n r^n$

Direct Comparison Test:

- When the given series, a_n looks like a known, or more simple, series, b_n

Limit Comparison Test:

- When you can see that the series looks like another convergent or divergent series, b_n
- But it is hard to say whether $b_n > a_n$ or $b_n < a_n$

Root Test:

- When the series can be written in the form: $\sum (a_n)^n$

Alternating Series Test:

- When the series can be written in the form: $\sum (-1)^{n+1} a_n$ or $\sum (-1)^n a_n$

Ratio Test:

- Whenever we are given something involving a factorial, e.g. $n!$
- Whenever we are given something involving a constant raised to the n^{th} power, e.g. $\sum \frac{n+5}{5^n}$

Integral Test:

- If the sequence is:
  - continuous
  - positive
  - decreasing (we can use the First Derivative Test here)

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Remember: These are just suggestions. There are other tests which may get us to the same answer.

Convergent	Divergent
Test for Divergence $\lim_{n\to\infty} a_n \neq 0$	Test for Divergence $\lim_{n\to\infty} a_n \neq 0$
$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$	$\sum_{n=1}^{\infty} \frac{1}{1}$
P-Series Test	P-Series Test
$\sum_{n=1}^{\infty} \frac{15}{3n^3}$	$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$
Geometric Series Test	Geometric Series Test
$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$	$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$
Direct Comparison Test	Direct Comparison Test
$\sum_{n=1}^{\infty} \frac{tan^{-1}(n)}{n\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$
Limit Comparison Test, Test for Divergence Fails $\lim_{n\to\infty} a_n = 0$	Test for Divergence Fails $\lim_{n\to\infty} a_n = 0$, Limit Comparison Test
$\sum_{n=1}^{\infty} \frac{1}{n^2+n}$	$\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+4}$
Root Test	Root Test
$\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$	$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$
Alternating Series Test	Alternating Series Test
$\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^2+n}$	$\sum_{n=1}^{\infty} (-1)^n \frac{1}{1}$
Ratio Test	Ratio Test
$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$	$\sum_{n=1}^{\infty} \frac{1}{1}$
Integral Test	Integral Test
$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$	$\sum_{n=1}^{\infty} \frac{1}{1}$